

PHOTON PROPAGATION IN THE CASIMIR VACUUM

JAVIER PARDO VEGA

Facultad de Física, Universidad de la Habana, Cuba
jpardo_v@ictp.it

HUGO PÉREZ ROJAS

*Grupo de Física Teórica, Instituto de Cibernética,
Matemática y Física, Cuba*
hugo@icmf.inf.cu

A transformation that relates the Minkowskian space of the Quantum Electrodynamics (QED) vacuum between parallel conducting plates and the QED vacuum at finite temperature is obtained. From this formal analogy, the eigenvalues and eigenvectors of the photon self-energy for the QED vacuum between parallel conducting plates (Casimir vacuum) are found in an approximation independent form. It leads to two different physical eigenvalues and three eigenmodes. We also apply the transformation to derive the low energy photons phase velocity in the Casimir vacuum from its expression in the QED vacuum at finite temperature.

I. INTRODUCTION

The propagation of light in vacuum modified by external conditions (background fields, finite temperature, boundary conditions) requires careful study since it may lead to conceptual changes such as the variation of the speed of light with regard to its value in the usual homogeneous and isotropic QED vacuum [1–5]. These modified vacua exhibit new properties such as a change in the dispersion equations and the energy density, with regard to the usual QED vacuum, leading to observable phenomena like the Casimir effect. The Casimir Effect is interesting in many fields of modern physics. For example, in astrophysics, gravitation and cosmology, it arises in space-time with a nontrivial topology. The polarization of vacuum due to the Casimir effect might be important for the resolution of the problem of the cosmological constant [6].

The propagation of soft photons through nontrivial QED vacua has been deeply studied, mostly within the framework of effective action. In particular, the investigations about light propagation in the vacuum between parallel conducting plates have lead to the so-called Scharnhorst Effect (superluminal phase and group velocities) [1, 2]. Latorre, Pascual and Tarrach proposed a unified formula for the low energy change in the averaged speed of light valid for the modified vacua, which was generalized by Dittrich and Gies [3, 5].

The analogy between the free energy in a finite temperature field theory and the Casimir energy for parallel planes under periodic conditions was firstly discussed by Toms [7]. On the other hand, the analogy between the Casimir vacuum and the QED vacuum at finite temperature under certain approximations was firstly pointed out by Barton [2] and later by Latorre et al. [3], and by E. Rodríguez Querts and one of the present authors (H.P.R.) [8]. In the last paper it is pointed out among other facts, a correspondence between Casimir vacuum and blackbody radiation energy densities, stemming from the fact that both problems involve the breaking of a space-time symmetry by a constant vector.

In the present paper, we exploit in a more advanced way the transformation and correspondence between Casimir vacuum and QED vacuum at finite temperature. We obtain the eigenvalues and eigenvectors of the photon self-energy in an approximation independent form. Based on this analogy we obtain the known expression for the low energy photons phase velocity in the Casimir vacuum at two-loop level and suggest an alternative interpretation to the resulting dispersion law.

II. CORRESPONDENCE BETWEEN THERMAL AND CASIMIR VACUA

In the imaginary time formalism of temperature, it is used Euclidean Field Theory, considered as an analytic continuation from Minkowskian space by the transformation in the time component of four-vectors $ct \rightarrow -i\tau$ ($x_0 \rightarrow -ix_4$). Then physical quantities are such that they are not translational invariant along τ , (but only under the finite shift $\tau \pm \beta$). In other words Euclidean "time" τ is restricted to the interval $[0, \beta]$, where $\beta = \frac{1}{T}$ where T is the temperature. According to it, fourth components of boson and fermion fields must satisfy periodic and antiperiodic boundary conditions with period β respectively.

In what follows we will call *thermal vacuum* to a medium at temperature T where a particle, for instance a photon, propagates in a background of electrons, positrons and other photons at temperature T . At the tree level, it corresponds

to blackbody radiation.

For the boson propagator we have

$$D_{\mu\nu}^T(\tau - \beta, \vec{x}; \tau', \vec{x}') = D_{\mu\nu}^T(\tau, \vec{x}; \tau', \vec{x}'), \text{ with } \tau, \tau' \in [0, \beta] \quad (1)$$

and for the fermion propagator

$$S_{\xi\eta}^T(\tau - \beta, \vec{x}; \tau', \vec{x}') = -S_{\xi\eta}^T(\tau, \vec{x}; \tau', \vec{x}'), \text{ with } \tau, \tau' \in [0, \beta]. \quad (2)$$

Here ξ and η are spinor indices and similar equations are valid for shifting τ' in β . The imposition of periodical conditions over one coordinate implies that its conjugate variable in Fourier space takes discrete values. Therefore, Matsubara discrete frequencies appear in momentum space.

Now, consider the vacuum between two infinitely extended, ideally conducting and neutral plates parallel to the 1, 2 plane at $x_3 = 0$ and $x_3 = a$, where a is the distance between plates. The electromagnetic field tensor obeys the following boundary conditions

$$n^\mu \tilde{F}_{\mu\nu}(x)|_{x_3=0,a} = 0, \quad (3)$$

where $n^\mu = (0, 0, 0, 1)$ is the four-vector whose spatial part is the normal vector of the plates and $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\beta\gamma}F^{\beta\gamma}$ is the dual field strength tensor. We will use the metric $g = \text{diag}(-, +, +, +)$ and we will take $\hbar = c = 1$ throughout the paper.

The photon propagator satisfying the boundary conditions (3) has been computed by Bordag et. al. and has been widely used by several authors [9]. Its expression is composed by the sum of two terms: the free propagator and a gauge-independent modification due to the boundaries. Let us make some reasonable approximations. The experimentally realized distances between plates are very large in comparison with the Compton wavelength of the electron. So, the limit $ma \gg 1$ holds for all realistic situations, where m is the electron mass. We will also consider that the external electromagnetic field vanishes near the plates. Therefore, in the momentum space the limit $ka \gg 1$ can be taken [1–3, 5]. Under the approximations $ma \gg 1$ and $ka \gg 1$ the photon propagator given by Bordag et. al. leads to the operative propagator derived by Latorre et. al., which corresponds to the photon propagator under periodic boundary conditions with period $2a$ for the spatial coordinate x_3 [3].

The imposition of boundary conditions on the Dirac field would alter the fermion propagator, but these changes decrease exponentially with distance from the plates (on a scale set by the Compton wavelength) and here we shall neglect them [1–3, 9]. Likewise, for cold heat baths ($T \ll m$), we can neglect the thermal contribution due to massive virtual particles which is always exponentially suppressed [5].

Then, the QED vacuum at low temperature and the vacuum between parallel conducting plates in the limit considered ($ma \gg 1$, $ka \gg 1$) are formally analogous, the last one reduces to impose boundary conditions on photons with period $2a$. Note that, in the Euclidean space, to do the transformation between the thermal and Casimir vacua it is enough to exchange the third and fourth components. Thus, the following transformation relates the Minkowskian spaces of both vacua:

$$x_0 \rightarrow -ix_3 \quad (4)$$

$$x_1 \rightarrow x_1 \quad (5)$$

$$x_2 \rightarrow x_2 \quad (6)$$

$$x_3 \rightarrow ix_0. \quad (7)$$

Here, it is done an analytic continuation in the components zero and three and the parameters β and $2a$ are identified. We will denote the transformation (4-7) as \mathcal{T} .

III. TENSOR STRUCTURE OF THE PHOTON SELF-ENERGY IN THE CASIMIR VACUUM

The exact photon propagator $D_{\mu\nu}$ can be expressed through the free photon propagator and the photon self-energy $\Pi_{\mu\nu}$. To find the Bose-excitation spectrum one must solve $D_{\mu\nu}^{-1}a^\nu = 0$ for the excitation field a^ν . Simultaneously the poles of the Green's function $D_{\mu\nu}$ are found. To fulfill this, it is convenient to diagonalize the photon self-energy [11]. Then, the Green's function is represented as

$$D_{\mu\nu}(k) = \sum_{i=1}^4 \frac{1}{k^2 - \kappa_i(k)} \frac{b_\mu^{(i)} b_\nu^{*(i)}}{b_\xi^{(i)} b_\xi^{*(i)}}, \quad (8)$$

where κ_i are eigenvalues and $b_\mu^{(i)}$ are eigenvectors of the photon self-energy, and the vector potential of the excitation is represented as

$$a_\mu(k) = \sum_{i=1}^4 \delta(k^2 - \kappa_i(k)) b_\mu^{(i)}(k). \quad (9)$$

Accordingly, to find the dispersion law for the wave whose vector potential is $b_\mu^{(i)}(k)$ one must solve the equation $k^2 = \kappa_i(k)$. One of the eigenvalues, say, with $i = 4$, is zero $\kappa_4 = 0$, $b_\mu^{(4)} = k_\mu$. This wave is purely longitudinal in the four-dimensional sense and the electromagnetic field strength for it is zero. So, it is enough to solve three dispersion equations.

Using the analogy with the thermal vacuum, the polarization tensor for the Casimir vacuum can be expressed in the Euclidean space as

$$\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\tilde{k}^2} \right) A(\tilde{k}^2, k_3^2) + \Pi_{33} \frac{k_\mu k_\nu}{\tilde{k}^2} \frac{k_3^2}{\tilde{k}^2} \quad (10)$$

$$\Pi_{\mu 3} = \Pi_{3\mu} = -\Pi_{33} \frac{k_\mu k_3}{\tilde{k}^2}, \text{ where } \mu, \nu = 1, 2, 4. \quad (11)$$

The Eqs. (10) and (11) have been obtained based on the structure of the photon self-energy for the thermal vacuum in the Euclidean space proposed by Fradkin [10]. This structure can be obtained by means of symmetry considerations.

The eigenvalues and eigenvectors of the photon self-energy for the Casimir vacuum in the Minkowskian space are

$$\kappa_1 = A \quad b^{(1)} = (k_1, k_0, 0, 0) \quad (12)$$

$$\kappa_2 = A \quad b^{(2)} = (0, k_2, k_0, 0) \quad (13)$$

$$\kappa_3 = \Pi_{33} \frac{\tilde{k}^2 + k_3^2}{\tilde{k}^2} \quad b^{(3)} = (k_0 k_3, k_1 k_3, k_2 k_3, -\tilde{k}^2). \quad (14)$$

The first and second modes are analogous to the transverse modes of the thermal vacuum, and the third mode is analogous to the longitudinal mode. For all modes, gauge invariance is preserved. The polarization directions of electric $\vec{e}^{(1,2,3)}$ and magnetic $\vec{h}^{(1,2,3)}$ fields in the waves with four-potentials $b_\mu^{(1,2,3)}$ are

$$\vec{e}^{(1)} = -i(k_1^2 - k_0^2, k_1 k_2, k_1 k_3) \quad \vec{h}^{(1)} = i k_0 (0, k_3, -k_2) \quad (15)$$

$$\vec{e}^{(2)} = -i(k_1 k_2, k_2^2 - k_0^2, k_2 k_3) \quad \vec{h}^{(2)} = i k_0 (-k_3, 0, k_1) \quad (16)$$

$$\vec{e}^{(3)} = -i k_0 k^2 (0, 0, 1) \quad \vec{h}^{(3)} = i k^2 (-k_2, k_1, 0). \quad (17)$$

For the first and second modes, the electric field is orthogonal to the vector $(k_1 k_3, k_2 k_3, -\tilde{k}^2)$ and the magnetic field is orthogonal to \vec{k} . Moreover, in both cases the electric field has a small component along the direction of \vec{k} : $\vec{e}^{(1)} \cdot \vec{k} = -i k_1 k^2$ and $\vec{e}^{(2)} \cdot \vec{k} = -i k_2 k^2$, which vanishes on the light cone mass shell $k^2 = 0$ and for propagation perpendicular to the plates. For the third mode, the magnetic field is along the 3-direction and the electric field is orthogonal to \vec{k} , however, both fields vanish on the light cone. The particular cases of propagation parallel and perpendicular to the plates have special interest and can be easily discussed from Eqs. (15-17). E.g., for propagation perpendicular to the plates, for the third mode $\vec{e}^{(3)} \parallel \vec{k}$ and the magnetic field is zero. Whereas, for propagation parallel to the plates, from the point of view of symmetry, the electric field corresponding to the first and second modes can have a small component along the direction of propagation which vanishes on the light cone.

Finally, the dispersion equation for the first and second modes is the same: $k^2 - A(\tilde{k}^2, k_3^2) = 0$ and for the third mode is $\tilde{k}^2 - \Pi_{33}(\tilde{k}^2, k_3^2) = 0$. The results of this section are general, determined by the symmetry of the system and independent of the order of approximation in the perturbative loop expansion. Let us use the transformation \mathcal{T} to obtain the known dispersion relation for the first and second modes of the Casimir vacuum from the dispersion relation for transverse modes of the thermal vacuum. The dispersion relation for the third mode of Casimir vacuum has not been computed in the literature.

IV. DISPERSION RELATIONS

In the limit $T \ll m$, thermal one loop effects are exponentially suppressed by the electron mass. Similarly, in the Casimir vacuum, one loop contributions to the dispersion relations due to the boundaries are exponentially suppressed.

However, the two-loop contribution involves a virtual photon within the fermion loop. In the low temperature regime the two loop contribution exceeds the influence of the one loop part due to the thermal excitation of the internal photon. Correspondingly, in order to obtain the radiative corrections to the dispersion relations in the Casimir vacuum one has to consider the two loop contribution.

By applying the transformation \mathcal{T} to the low energy photons dispersion equation for transverse modes of the thermal vacuum [2], we obtain the dispersion equation for modes one and two in the Casimir vacuum

$$\omega^2 = k^2 \left(1 + \alpha^2 \frac{11\pi^2}{4050} \frac{1}{a^4 m^4} \cos^2 \theta \right), \quad (18)$$

where θ is the angle between the direction of propagation and the normal to the plates. Hence, computing the phase velocity we arrive to the Scharnhorst's result $v = \frac{\omega}{|\vec{k}|} = 1 + \frac{11\pi^2}{8100} \alpha^2 \frac{1}{a^4 m^4} \cos^2 \theta$. The phase and group velocities coincide and, according to the standard interpretation, are greater than c for propagation perpendicular to the plates [4].

However, the breaking of translational invariance in Casimir vacuum, leads to non-conservation of momentum along the 3-direction. Thus we suggest an alternative interpretation of the dispersion relation of the first and second modes by understanding (18) as meaning an effective photon momentum in the 3-direction $k'_3 = \left(1 + \frac{11\pi^2}{8100} \alpha^2 \frac{1}{a^4 m^4} \right) k_3$, conserving the dispersion relation $\omega = |\vec{k}|$. This last interpretation is consistent with the fact that the second order corrections to the Casimir force are repulsive.

V. CONCLUSIONS

We found an explicit transformation (\mathcal{T}) relating Minkowskian thermal vacuum and Casimir vacuum spaces. Based on this transformation, the structural properties of the photon self-energy and the polarizations of eigenmodes were described. For eigenmodes one and two the electric field has small component along \vec{k} which vanishes on the light cone and the magnetic field is orthogonal to \vec{k} ; while both fields in the third mode vanish on the light cone. The dispersion relation for low energy photons for the first and second modes in Casimir vacuum within the two loop approximation can be re-obtained based on the analogy with the thermal vacuum and we proposed an alternative interpretation keeping photons on the light cone.

Acknowledgements

The authors thank the OEA-ICTP for its support through NET-35.

-
- [1] K. Scharnhorst, *Phys. Lett. B* **236** (1990) 354.
 - [2] G. Barton, *Phys. Lett. B* **237** (1990) 559.
 - [3] J. Latorre, P. Pascual and R. Tarrach, *Nucl. Phys. B* **437** (1995) 60.
 - [4] K. Scharnhorst, "The velocities of light in modified QED vacua", arXiv:hep-th/9810221v2, 1998.
 - [5] W. Dittrich and H. Gies, *Phys. Rev. D* **58** (1998) 025004.
 - [6] M. Bordag, G.L. Klimchitskaya, U. Mohideen, V.M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, USA, 2009).
 - [7] D.J. Toms, *Phys. Rev. D* **21** (1980) 928.
 - [8] H. Pérez Rojas and E. Rodríguez Querts, *Int. J. Mod. Phys. A* **21** (2006) 3761.
 - [9] M. Bordag, D. Robaschik and E. Wieczorek, *Ann. Phys.* **165** (1985) 192.
 - [10] E. S. Fradkin, in *Quantum Field Theory and Hydrodynamics — Proc. P.N. Lebedev Phys. Inst.*, ed. D. V. Strobelt'syn (Consultants Bureau, New York, 1967).
 - [11] H. Pérez Rojas and A. E. Shabad, *Ann. Phys.* **121** (1979) 432.